Mathematical analysis of physical models consists in showing that they are well-posed, i.e. they admit a unique solution, in a well-suited functional framework. The notion of solution can change according to the nature of the nonlinearities in the model. We have to study the particular properties of the solution such as its regularity or its long-time asymptotic behaviour. Moreover spectral analysis of the models is very useful since it supplies a lot of informations about the behaviour of the system such as instability growth rates and spatial structure of the instable global eigenmodes.

For examples we have obtained a set of results about the derivation and the mathematical analysis of waterbag-type models in different physical configurations. Waterbag models come from the exact geometric reduction of the Vlasov equation thanks to geometric Liouville invariants and their mathematical structures are closely related to hyperbolic systems of conservation laws, in finite or infinite dimension with non-local fluxes [7,10,11,13]. Regarding to spectral analysis we have performed a numerical study of the eigenvalue problem for the gyrowaterbag model in cylindrical geometry [16,17]. An asymptotic and spectral analysis of the gyrowaterbag integrodifferential operator in toroidal geometry has led to the development of a parallel numerical code whose results are very promising and in good agreement with those of a quasilinear code [14].

In addition we often have to build and justify new reduced models, starting from original base ones, in order to take into account their own properties. This modeling effort is necessary to reduce the computational cost of original base models while it allows to retain the essentiel physics of the studied phenomenon.

For example Using an Eulerian variationnal principe of least action (Euler-Poincaré), asymptotic perturbation methods and Lagrangian averaging techniques we have obtained a system of PDEs called « Lagrangian averaged gyrowaterbag continuum » (LAGWBC) designed to accurately capture the dynamics of the turbulent plasma flow at length scales larger than a typical length scale while averaging the motion at scales smaller than this typical length scale. In the isotropic setting, we have shown that this system has unique strong solutions locally in time. Regarding to the original gyrowaterbag continuum, the LAGWBC equations show some additional properties and several advantages from the mathematical (regularizing terms, well-posedness) and physical (modeling of small scales, anisotropic tensor of turbulent fluctuations, dispersion nonlinear terms) viewpoints, which make this model a good candidate for describing accurately gyrokinetic turbulence in magnetically confined plasma [15].

We also aim at performing a numerical analysis of the physical models. In other words, we design efficient numerical schemes for solving nonlinear PDEs (kinetic and multi-fluid transport equations, elliptic and waves equations). We also make the mathematical analysis of these numerical schemes, i.e. we prove in a well-suited mathematical framework, the convergence of the approximated numerical solution of the discrete problem towards the exact solution of the continuous problem and obtain a priori error estimates.

For examples we have obtained set of results about the mathematical proof of the convergence of very high-order semi-lagrangian schemes and a priori error estimates for kinetic equations such as Vlasov-Poisson [4,5] and Vlasov-Einstein systems [9]. We have also performed the construction and make the analysis of
discontinuous-Galerkin schemes which preserve the incompressible constraint of the magnetic field for the induction equations of MHD system [2].

Finally we develop efficient parallel numerical schemes for a large number of processors, which becomes essential because of the high dimensionality of kinetic models.

For examples we have designed new efficient semi-lagrangian parallel schemes on fixed grids, unstructured meshes and adaptive meshes of phase-space from one to five dimensions for Vlasov-type equations (Vlasov-Poisson, Vlasov-Maxwell, Vlasov-Darwin, Vlasov-ondes, Vlasov-gyrokinetic) [1,3,6]. We have also developed semi-lagrangian and discontinuous-Galerkin schemes for reduced models of waterbag-type (gyrowaterbag, waterbag-{Poisson, Maxwell, quasineutral}) which are multi-fluid models of one to three space dimensions and are similar to hyperbolic systems of conservation laws [7,8,16,17].

Below we give an example of the simulation of the 3D gyrowaterbag model (GMWB3D-SLCS code [8,16]) with 12 3D-contours of the 4D phase-space. The figures depict the density perturbation of the plasma which is obtained as a sum over all the contours. The test case below deals with the development of an ion-temperature-gradient instability driving the plasma to a turbulent state.


[17] D. Coulette, N. Besse, Multi-water-bag models of ion temperature gradient instability in cylindrical geometry. Submitted